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# Mathematics Fundamentals

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## Index

**General objectives.**

The general objectives of training in " Mathematics Fundamentals "are:

1. Understand some basic concepts and concepts of mathematics.
2. Take advantage of the logical, methodical, and organized nature of mathematics.
3. Create or strengthen sustained bases for the future assimilation of higher-level mathematics knowledge.

**Introduction.**

It is intended to approach, remember, or deepen some basic knowledge of mathematics. It addresses the actual numbers, equations and Inequalities, functions and respective domains and zeros, and a very important set of functions that are polynomials. Finally, two important functions: exponential and logarithmic.

In addition to being necessary in the process of entering higher education through the system of higher 23 years, this learning is essential as a way of remembering important basic knowledge for curricular units, in higher courses, "Mathematics", "Mathematics I" and "Mathematics II".

Good study!

## 1. Sets and ranges of real numbers.

In most exercises, you will work with real numbers. More specifically, with the set of real numbers, **R.**

### 1.1. Sets.

A set can be seen as a collection of objects of any nature. Objects are the elements of the set. When elements are numbers the set of numbers appears.

**Examples of numbers are as follows:**

1. **Natural Numbers** - sum of the real 1 to itself successively. The set of natural numbers is called **N** : 1, 2, 3,
2. **Integers** - Zero and all positive and negative integers; The set of integers is called **Z.** -3,-2,-1,0,1,2
3. **Rational Numbers** – Numbers that can be expressed as a quotient a/b, where a and b are integers, b other than 0. The set of rational numbers is called **Q**. -3, -2, -1, 0, 2/3, 1, 5/4, 2
4. **Irrational number** - Non-rational real number (PI = 3.141592664 ; 1/3=0.3333333).

The **set of the actual numbers** is represented by the symbol **R** and encompasses all the numbers of the previous sets.

**1. Representation on the real line.**

The real numbers can be represented on a line: **the actual line**. On this line there is a number (zero) that between negative numbers and positive numbers. The entire line represents the set of real numbers. If only a portion of the line is chosen, this represents a subset of **R.** These subsets of real numbers can also be represented by **ranges of real numbers.**

**Like this:**

- **[1,2]** represents all real numbers greater than and equal to 1 and simultaneously less than or equal to 2. That is, all real numbers between 1 and 2, including 1 and 2. It can also be said that the interval is closed at both ends.
- **[1,2[ represents** all real numbers greater than and equal to 1 and simultaneously less than 1. That is, all actual numbers between 1 and 2, including 1 and excluding 2. It can also be said that the range is closed to the left and opened on the right.

## 2. Equations or equalities

### 2.1 Equations or equalities.

Recalling the concept **equation (equality).**

$2x + 4 = 20$

This is a simple example of an equation containing a variable, but that is extremely useful and appears in most situations. It can be observed that, in the mathematical writing of an equation, there is:

1. one or more letters indicating unknown values, which are called variables or unknowns. The letter x is unknown.
2. a sign of equality, denoted by =;
3. an expression to the left of equality, called the first member or member of the left.
4. an expression to the right of equality, called second member or member of the right.

This equation can be solved by originating the solution of $x=8$. Note the reading that can be made:

1. replacing x by 8, you get $16+4=20$, $20=20$. It is a true equality.
2. the real number 8 is the only number, the only solution, that makes equality true. If any other real number is used, the resulting equality will be false.
3. before viewing an equation such as $2x+4=20$, the student should ask himself the following question: what or what are the real numbers that make equality true? To answer this question, the equation is solved.

### 2.2 Inequalities.

Now moving on to **inequality.** How to identify an inequality?

**First-degree** inequalities (also called inequalities) are mathematical expressions, with unknowns, in which the terms (left-hand member and right-sided limb) are linked by one of the four signs:

1. < *which means* minor.
2. > *means* greater.
3. <= which *means less* than or equal.
4. >= that *means greater* or equal.

In **inequalities**, the goal is to obtain a set of numbers (the solution of the problem, the subset of R) that make their inequality true.

We can simply transform the previous equation into an inequality by introducing one of the four above-mentioned signals. By way of example:

$2x+4 <= 20.$

When observing this exercise, the student should again ask himself "what or what are the real numbers that make inequality true? To answer this question, the inequality is resolved. Resolution:

$2x+4<=20 \Leftrightarrow$

$2x<=20-4 \Leftrightarrow$

$2x<=16 \Leftrightarrow$

$x<=8.$

That is, any number less than or equal to 8 values makes inequality true and is therefore part of the exercise solution. The solution, in the form of an interval, would be presented as **S=]- oo, 8]**.

And if the inequation to be solved is of the kind $(x+3)/(x+8)>9$.

How to solve this equation? It is suggested the adoption of a robust method, in which a table is used to research the signs (positive and negative) of the function. To reach this table, the necessary and valid mathematical manipulations must be performed for the right member of the inequation to appear the number zero. Zero has a very particular characteristic: it is the actual number that distinguishes the positive numbers from the negative numbers. That is, if inequation gets in shape:

The left-hand member >0 means that the left-sided member is positive or by which to look for the subset ***of R*** for which the "plus sign" appears on the board.

# 3 Functions and equations.Linear functions and equations.

## 3.1.1. Linear Equations

As a particular case of the various types of equations, there is the group of linear equations, whose generic equation can be represented by:

$y = m*x + b$, em que *m* e *b* são constantes (números reais).

This group of equations represents, in graphical terms, the equation of a line, *in which m* is the slope and *b* a ordered at the origin.

## 3.1.2. Functions

One of the most important concepts of mathematics is the concept of function. And what is a function?

It's a law of transformation. And what does it turn into? It turns numbers into other numbers. That is, a certain number corresponds to another number.

The "starting number" is usually called original or object, and the number that is its correspondent is usually called the image.

If you join in a bag, all the originals, we form a set of numbers. If you join, in another bag, the images, we form another set of numbers.

As mentioned above, in the curricular units of "Mathematics" we work with real numbers: the set ***R***.

The following figure illustrates the appearance of a function (using the balloon diagram).

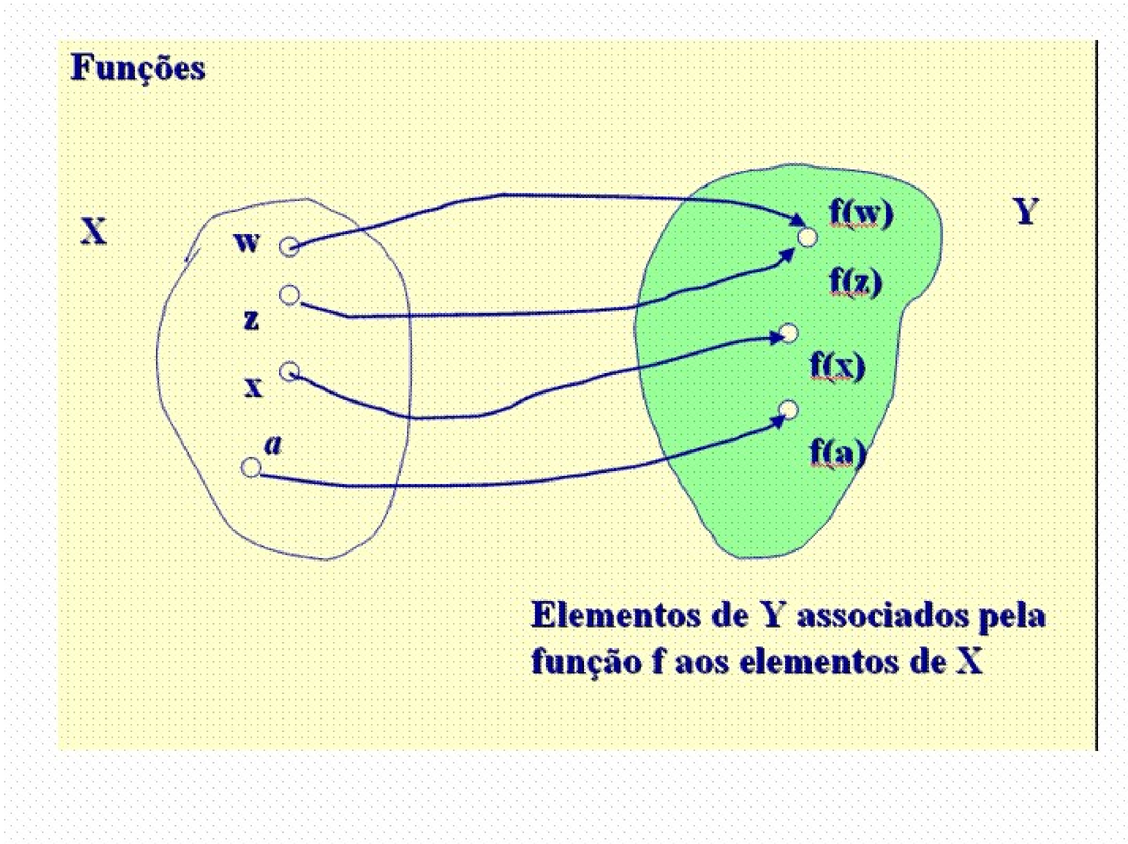


**Figure 1** - Schematic representation of the concept of function.

**VERY IMPORTANT:** each "starting" number (balloon on the left side) can only match one and only one number from the second set (balloon on the right side), so that this transformation law can be called a function.

### 3.1 Real variable real function domains and zeros

#### 3.2.1. Function Domain

To the set of numbers that contains all the originals of a function is called a function domain. It is represented by ***Df***.

The set of numbers consisting of all images is called counter domain of a function and is represented by ***D'f***.

According to the previous point (3.1), an original can only match one image. But two different originals can have the same image. What matters, to be function, is that each document has only one image. If it is equal to the image of another original, the law of transformation continues to be able to be classified as a function.

The originals are usually represented by *the letter x*, while the images by the letter *y*. Like this, *y=f(x).* In this symbolic notation, we observe the transformation of the *number x* into the number *y* through the*f*.

There is a function f(x), the determination of the domain always implies the verification of the possibility that all the actual numbers can, according to this function, have the corresponding image. It is in this basic idea that the calculation should be made. That is, a number can only belong to the domain of a function if, through the law of transformation, another number has been given.

In calculating the domain, some important particular cases should be taken into account:

1. a function with denominator. In this case, the condition to place in the domain calculation is that the denominator must be nonzero.
2. a function with a root. In this case, it should be known that if the root is odd index can always be calculated, and there is no problem or restriction in the calculation of the domain. If the root is index pair, the condition to consider, in the domain calculation, is that what "is under the root" has to be greater than or equal to zero.

If a root appears in the denominator, both situations should be analyzed simultaneously.

#### 3.2.2. Zero function.

It is a very simple concept: the zero of a function is the original whose image is the number zero. Therefore, the equation arises y = 0, ou seja, f(x) = 0.

### 3.2 Polynomials functions and quadratic functions

Be the following function, $f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots a_n$

This function is a polynomial of grade N. It is a polynomial function. The "a" are the coefficients of the polynomial and are real numbers.

If n=2, the polynomial becomes $f(x) = a_0x^2 + a_1x^1 + a_2$

This function is a polynomial of the 2nd degree or also called quadratic function.

To find the roots or zeros of a quadratic function can be used the resolvent formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A concrete example of a 2nd degree polynomial: $f(x) = x^2 + 2x + 1$

Exercise 1: Determine the roots of $f(x) = x^2-3x+2$.

Answer: x=2 or x=1.

### 3.3 Polinomial decomposition

The interesting thing about this process of root determination of a polynomial of the 2ºgrey, is that $x^2-3x+2$ can be written as being (x-2)(x-1). That is, $x^2-3x+2 = (x-2)(x-1)$. Check.

A polynomial of any degree can be equal to the product of a set of factors that include the roots or zeros of it. That is, it can be decomposed into a product of factors!

For example, 10 = 5*2. That is, 10 is equal to the product of 5 times 2. It's two factors.

On the other hand, it can also be written that 5 = 10/2. That is, one of them obtained through a division. It's the same three numbers.

The same reasoning can be done with polynomials. In the case of the example above, for example, x-1= ($x^2$-3x+2)/(x-2).

That is, through a division of two polynomials it is also possible to proceed to a decomposition of the polynomial.

If one of the roots of the polynomial is known, that is, the factor (x- root), you can use a split to help decommission the polynomial.

Notice the next split:

$$\begin{array}{l|l} \cdots x^2 + x - 6 & \underline{x+3} \\ \underline{-(x^2+3x)} & x-2 \\ \underline{\cdots 0 - 2x - 6} & \\ \underline{-\cdots(-2x-6)} & \\ 0 & \end{array}$$

As we know, the dividend is equal to the divisor times the quotient plus the rest. That is, and in the specific case, $x^2$+x-6 = (x+3)(x-2)+0=(x+3)(x-2)

The polynomial has been

decomposed!

## 3.4 Exponential and logarithmic functions

Remember that:

$$a^x a^y = a^{x+y}$$

$$(a^x)^y = a^{xy}$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

### 3.4.1 Exponential function

It is a law of transformation of the kind $f(x)=a^x$, in that

*a* - constant and positive real number and is the basis of the exponential function

The exponential function f(x) is always positive (that is, the counterdomain is a set of positive numbers).

Most used bases:

- *a = 1;*
- *a = e* (natural exponential function).

Depending on the numerical values for the base "a", there are two types of charts, as can be seen from the following figure.

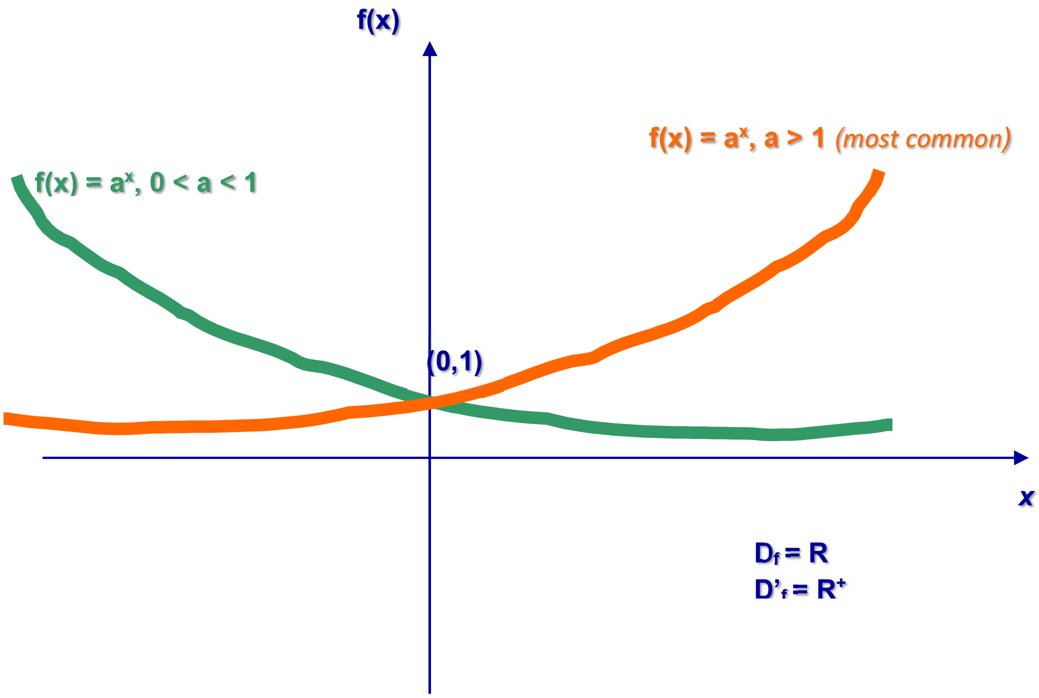


**Figure 2** - Graphical aspect of the exponential function.

### 3.4.2.Logarithmic function

The logarithmic function is the inverse function of the exponential function. That is, the numbers in the pair (original, image) that represents all the points of the line of the function graph, will change position when you move to the reverse function.

For example, in the exponential function (see previous graph), whatever the basis, for the original zero the image is number one. The point on the chart is (0.1). From the above information, the logarithmic function graph will appear the point (1.0), whatever the basis.

Symbology associated with basic logarithmic function *a:*

$f(x) = \log_a(x)$, $a > 0$ e $a \neq 1$

Df = $R^+$ (counterdomain of exponential function)

Considering the above information, the following logarithm definition of a number arises:

Logarithm *of a positive number x* on the basis *a* is the number (*y*) to which to raise the base to *obtain x.*,

$$f(x) = y = \log_a(x) \Leftrightarrow x = a^y$$

**Operative properties of logarithms**

$$\log_a(x * y) = \log_a x + \log_a y \qquad \forall\, x, y \in \Re^+, \forall\, a \in \Re^+ \setminus \{1\}$$

$$\log_a (x / y) = \log_a x - \log_a y \qquad \forall\, x, y \in \Re^+, \forall\, a \in \Re^+ \setminus \{1\}$$

$$\log_a x^p = p \log_a x \qquad \forall\, p \in \Re, \forall\, x \in \Re^+, \forall\, a \in \Re^+ \setminus \{1\}$$

$$\log_b x = \log_a x * \log_b a \qquad \forall\, x \in \Re^+, \forall\, a, \mathrm{b} \in \Re^+ \setminus \{1\}$$

The graph corresponding to the logarithmic function is shown in the following figure.

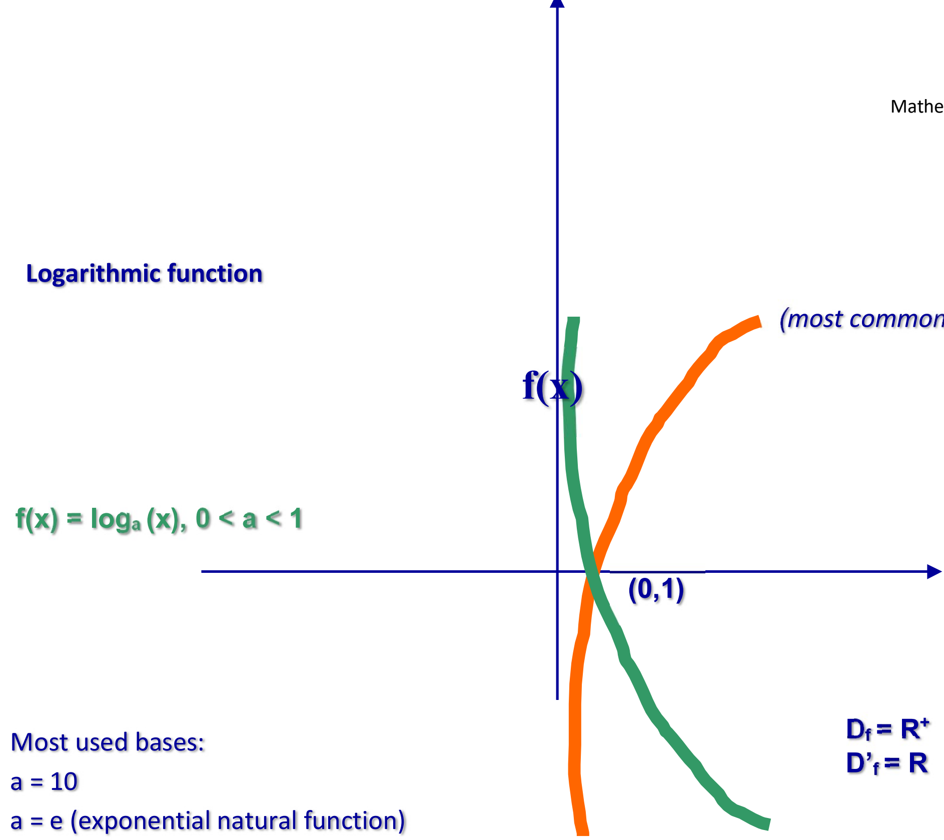


**Figure 3** - Graphic aspect of the logarithmic function.

A possible exercise associated with these functions is:

$$y = \log_{10}(1000)$$

What is the value of y?

The answer is y=3, because 3 is the number to which to raise the base to get the number 1000. It is added that in this concrete example, and for the logarithmic function presented, the original is the number 1000, its image is the number 3, being the point of the graph (1000, 3).

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